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# BENDING BEHAVIOUR OF FUNCTIONALLY GRADED METAL-CERAMIC TRIANGULAR PLATE

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## ABSTRACT

The bending behaviour of functionally graded triangular plate is studied using the finite element method based on first order shear deformation theory. Material properties are assumed to be graded along the thickness direction according to a simple power law distribution in terms of volume fractions of the constituents. An eight noded isoparametric element (with six degrees of freedom at each node) is considered for meshing and analysis. Results are presented for variation of deflection and normal stresses in functionally graded triangular plates with simply supported and clamped boundary conditions subjected to uniform normal pressure.

**Keywords**: Finite element method, first order shear deformation theory and functionally graded triangular plate.

## I. INTRODUCTION

Laminated composite materials are widely accepted for various engineering applications. Though laminated composite materials posses high strength-weight ratio and stiffness-weight ratio by laminating one material over another, the sudden discontinuity of material causes stress concentration and de-lamination.

To minimize the stress concentration and de-lamination, a new class of composite materials called Functionally Graded Materials (FGM's) were introduced by a group of material scientists in Japan around 1984. Unlike laminated composite materials, the material properties in functionally graded materials vary smoothly and continuously from one layer to another through the thickness resulting in lesser stress concentration, residual stresses and de-lamination.

FGM's made of metal-ceramic combination are most widely used in aircraft structures, space vehicles, nuclear reactors and other high thermal application areas as ceramic offers high resistance to temperature change (thermal loads) and metal provides strength, stiffness and fracture toughness to resist the mechanical loads. FGM's have great scope in civil engineering structural members such as beams, plates and shells as they can withstand thermo-mechanical loads and offer high fire resistance.

## II. BRIEF REVIEW OF LITERATURE

The earlier studies on bending of isotropic plates of rectangular, circular, and triangular plates were based on application of classical mechanics by Timoshenko and Krieger [1]. Static behaviour of functionally graded rectangular plates based on third-order shear deformation plate theory was studied by Reddy [2]. Cheng and Batra [3] presented the deflections of a simply supported functionally graded polygonal plate by the first-order shear deformation theory and a third-order shear deformation theory to that of an equivalent homogeneous Kirchhoff plate. Vel and Batra [4] have presented an exact 3-D solution for the thermo-elastic deformation of functionally graded simply supported plates of finite dimensions. Reddy and Chin [5] performed thermo-mechanical studies on functionally graded plates. Zenkour [6] presented comprehensive studies on functionally graded sandwich plates. Zenkour [7] analyzed the functionally graded plates by generalized shear deformation theory. Chakraverty and Pradhan [8] have conducted extensive studies on free vibration of triangular functionally graded plates.

From the brief literature presented above, it can be observed that lot of research on functionally graded rectangular and square plates were done. From the above literature review it can be understood that a very few studies on FG triangular plates are present. No study on flexural behaviour of FG triangular plates is present. Hence, in this study flexural behaviour of functionally graded triangular plates is investigated.

## III. VALIDATION

For validating the analysis by ANSYS, a FG square plate analyzed by previous researcher [7] was reanalyzed and the obtained results were tabulated in Table 1 along with the results of previous study. A total number of 61 elements were used for meshing. The effect of volume fraction on deflection and stresses can be observed from the Table [1]. It can be observed from the Table 1 that the obtained results are in good agreement with the results of previous study.

*Table 1 Validation of results*

| n | Study | $W^*$ | $\sigma_x^*$ | $\sigma_z^*$ |
|---|---|---|---|---|
| 1 | Present | 0.969 | 4.687 | 2.541 |
| | Ref. [8] | 0.9287 | 4.4745 | 2.1692 |

Where n = volume fraction exponent
$W^*$ = Normalized deflection along Y axis
$\sigma_x^*$ = Normalized stress along X axis
$\sigma_z^*$ = Normalized stress along Z axis

## IV. MODELING AND ANALYSIS

Functionally graded triangular plate modeled and analyzed using ANSYS. An equilateral triangle is considered. Meshing is done by using SHELL 281 element. The element is capable of analyzing thin and moderately thick layered plates and shells. The element has eight nodes with six degrees of freedom at each node. Finite element analysis of plate is done based on First Order Shear Deformation Theory (FSDT). The image of the element is shown in Figure 1.

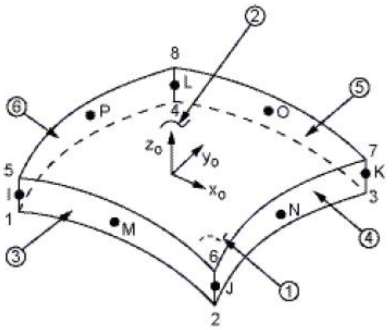

*Figure 1 SHELL 281*

It is assumed that top layers are rich in ceramic content and bottom layers are rich in metal content as shown in Figure 2.The analyses are performed for thin plates (a/h=100) and moderately thick plate (a/h=50). A total number of 64 elements are used for meshing. The material properties are as follows $E_c$ = 151 GPa; $E_m$ = 70 GPa and ν = 0.3; where $E_c$ and $E_m$ are modulus of elasticities of ceramic and metal respectively. A total number of 30 isotropic layers are modeled such that the variation of material property that of FGM is achieved. Both simply supported (u = v = w

= 0 at all edges) and clamped ($u = v = w = M_x = M_y = M_z = 0$ for all edges) boundary conditions are considered. A uniform lateral pressure ($P_o$) of 1000 N/m$^2$ is applied on to the triangular plate. The modeled image of the triangular plate is shown in Figure 3.

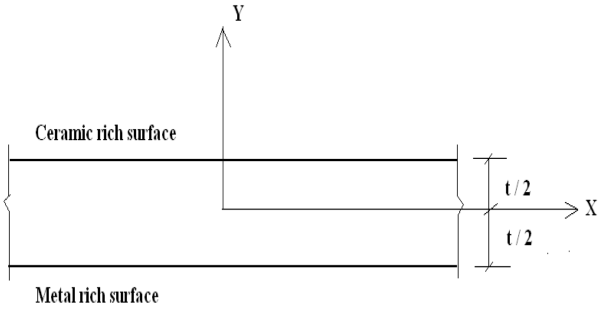

*Figure 2 Material distribution along thickness of plate*

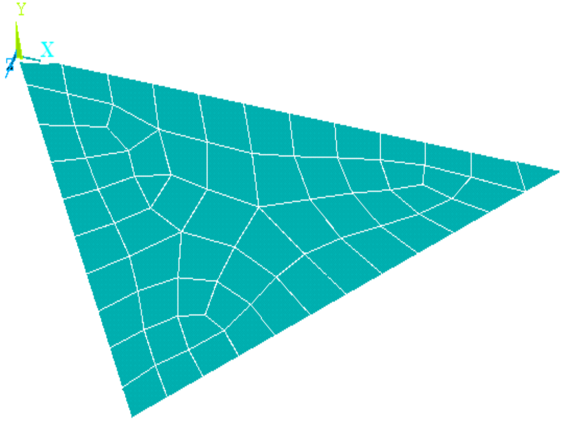

*Figure 3 Modeling and meshing of triangular plate*

## V. RESULTS AND DISCUSSION

By performing finite element analysis of FG triangular plate by ANSYS, the obtained normalized deflections and normal stresses are presented in Tables 2 and 3. Normalized deflection and stresses are obtained from Equations (1) – (3).

$W^* = (10t^3 E_c / a^4 P_o)$ (1)

$\sigma_x^* = (t/aP_o)\, \sigma_x$ (2)

$\sigma_z^* = (t/aP_o)\, \sigma_z$ (3)

Where $W^*$ = Normalized deflection in Y direction
$\sigma_x^*$ = Normalized normal stress in X direction
$\sigma_z^*$= Normalized normal stress in Z direction
$\sigma_x$ = dimensional normal stress in X direction
$\sigma_z$= dimensional normal stress in Z direction
t = total thickness of the plate
$E_c$ = modulus of elasticity of ceramic material
a = length of side of triangular plate
$P_o$ = applied uniform pressure

*Table 2 Normalized deflection and normal stresses for simply supported FG triangular plate*

| **(a/h) ratio** | **n** | **$W^*$** | **$\sigma_x^*$** | **$\sigma_z^*$** |
|---|---|---|---|---|
| 100 | 1 | 0.0865 | 15.7 | 14.9 |
| | 2 | 0.0939 | 16.8 | 15.9 |
| | 3 | 0.0975 | 17.3 | 16.4 |
| 50 | 1 | 0.0878 | 7.92 | 7.604 |
| | 2 | 0.0952 | 8.466 | 8.116 |
| | 3 | 0.0991 | 8.72 | 8.382 |

*Table 3 Normalized deflection and normal stresses for clamped FG triangular plate*

| **(a/h) ratio** | **n** | **$W^*$** | **$\sigma_x^*$** | **$\sigma_z^*$** |
|---|---|---|---|---|
| 100 | 1 | 0.0252 | 7.195 | 8.398 |
| | 2 | 0.0276 | 7.828 | 9.138 |
| | 3 | 0.0285 | 8.077 | 9.430 |
| 50 | 1 | 0.0259 | 3.63 | 4.24 |
| | 2 | 0.0283 | 3.95 | 4.61 |
| | 3 | 0.0294 | 4.072 | 4.754 |

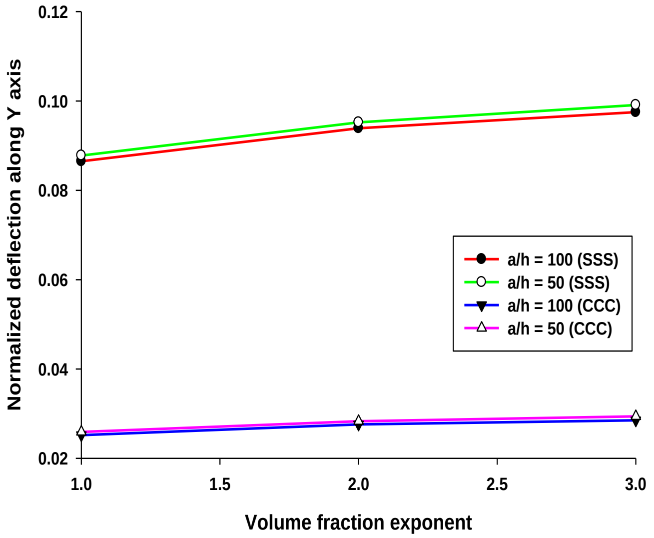

*Figure 4 Normalized deflection ($W^*$) versus Volume fraction exponent (n)*

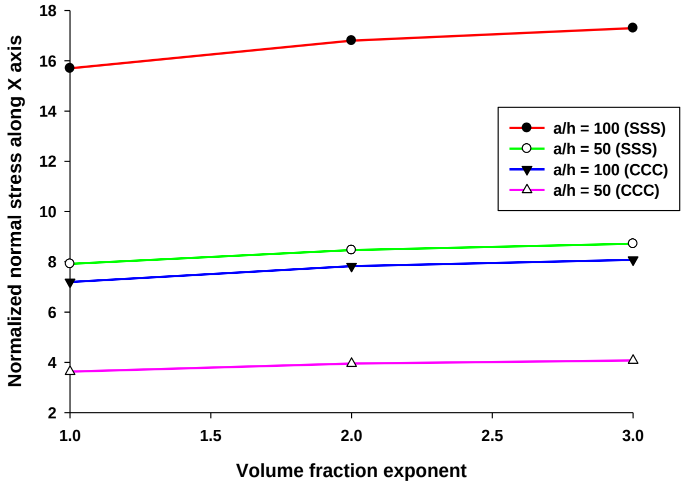

*Figure 5 Normalized normal stress ($\sigma_x^*$) versus Volume fraction exponent (n)*

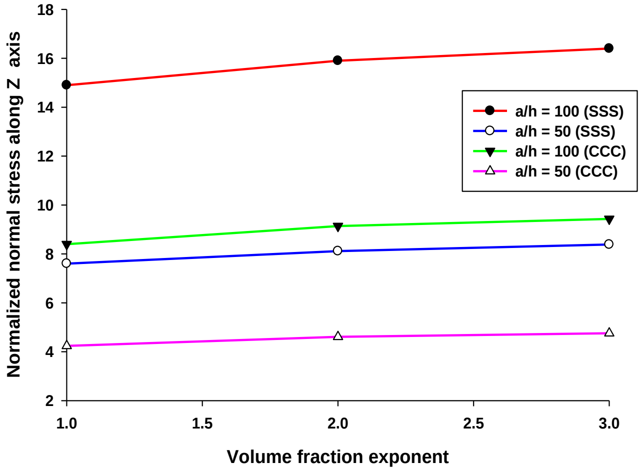

*Figure 6 Normalized normal stress ($\sigma_z^*$) versus Volume fraction exponent (n)*

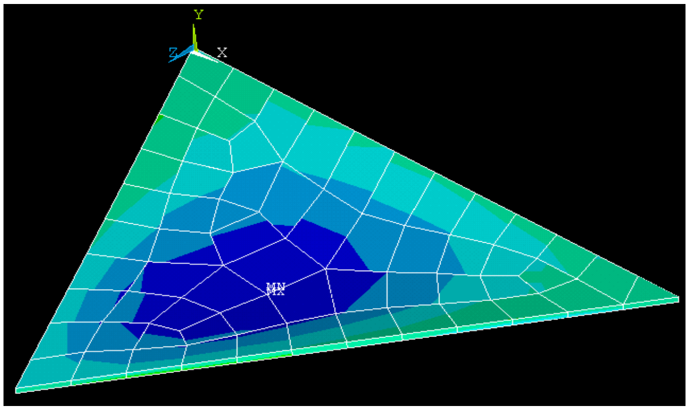

*Figure 7(a) Normal stress along X axis*

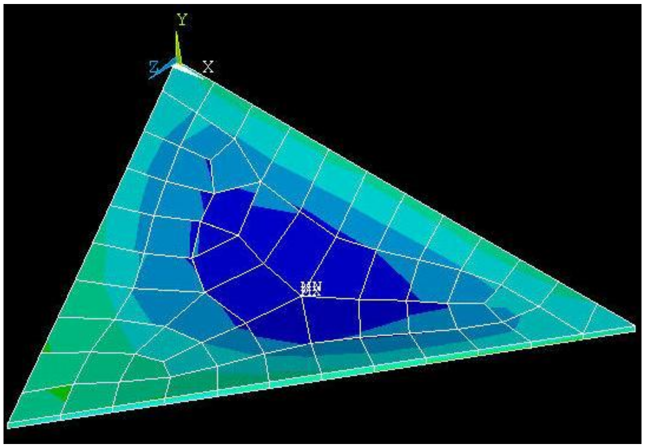

*Figure 7 (b) Normal stress along Z axis*

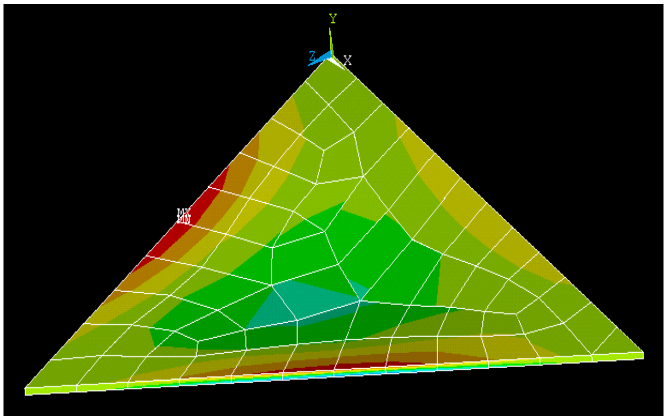

*Figure 8(a) Normal stress along X axis*

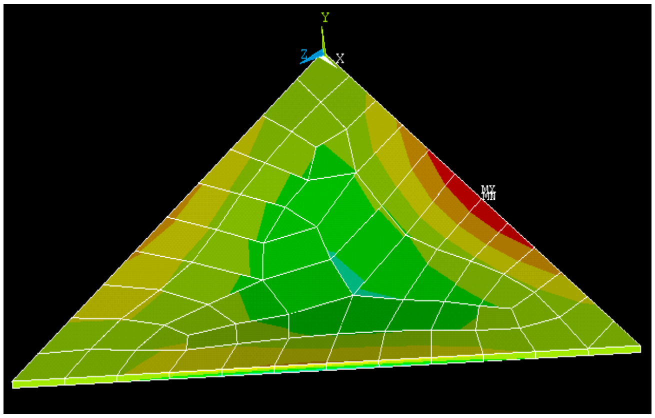

*Figure 8 (b) Normal stress along Z axis*

Figures 4, 5 and 6 represent the effect of volume fraction exponent on deflection, normal stress - $\sigma_x^*$ and normal stress – $\sigma_z^*$ respectively for both simply supported and clamped moderately thick and thin FG triangular plates. Figures 7 and 8 represent the location of minimum and maximum normal stresses (at top and bottom) for simply supported and clamped boundary conditions.

From the Tables 2 and 3 and Figures 4-8, the following may be observed

- Thin plates (a/h = 100) resulted in lesser normalized deflections than the moderately thick plates (a/h = 50) for both simply supported and clamped boundary conditions.
- Thin plates (a/h = 100) resulted in greater normalized normal stresses ($\sigma_x^*$ and $\sigma_z^*$) than the moderately thick plates (a/h = 50) for both simply supported and clamped boundary conditions.
- Increase in volume fraction exponent (n) resulted in increase of normalized deflections for both thin and moderately thick plates considering both simply supported and clamped boundary conditions.
- Increase in volume fraction exponent (n) resulted in increase of normalized normal stresses ($\sigma_x^*$ and $\sigma_z^*$) for both thin and moderately thick plates considering both simply supported and clamped boundary conditions.
- For a particular value of volume of fraction exponent (n) and a/h ratio of plate, clamped plates resulted in lesser normalized deflection and normal stresses when compared to simply supported plates.
- From Figure 7, it can be observed that the minimum and maximum normal stresses ($\sigma_x$ and $\sigma_z$) occurred at top and bottom of the plate respectively within the centre zone of the simply supported triangular plate.
- From Figure 8, it can be observed that the maximum and minimum normal stresses ($\sigma_x$ and $\sigma_z$) occurred at top and bottom of the plate respectively at the edges of the clamped triangular plate.
- The normal stresses are compressive at top and tensile at bottom throughout the entire plate for simply supported boundary condition.
- For the plates with clamped boundary condition, the normal stresses are tensile at top and compressive at bottom near the clamped edges of the plates and are compressive at top and tensile at bottom near the centre zone of the plate

## VI. CONCLUSION

In this investigation, the bending behaviour of functionally graded triangular plate is studied by using finite element method based on first order shear deformation theory. Eight noded shell element (SHELL 281) is considered for meshing and analysis. Results are presented for the variation of deflection and stresses in triangular plate with simply supported and clamped boundary conditions when subjected to uniform normal pressure. From the study the following conclusions may be drawn

- Thin plates resulted in lesser normalized deflections and greater normalized normal stresses for both simply supported and clamped boundary conditions when compared with moderately thick plates.
- Increase in volume fraction exponent resulted in increase of normalized deflections and stresses for both thin and moderately thick plates for both simply supported and clamped boundary conditions.

- For a particular value of volume of fraction exponent (n) and a/h ratio of plate, clamped plates resulted in lesser normalized deflection and normal stresses when compared to simply supported plates.
- Minimum and maximum normal stresses ($\sigma_x$ and $\sigma_z$) occurred at top and bottom of the plate respectively within the centre zone of the simply supported triangular plate. Maximum and minimum normal stresses ($\sigma_x$ and $\sigma_z$) occurred at top and bottom of the plate respectively at the edges of the clamped triangular plate.

Normal stresses are compressive at top and tensile at bottom throughout the entire plate for simply supported boundary condition. For the plates with clamped boundary condition, the normal stresses are tensile at top and compressive at bottom near the clamped edges of the plates and are compressive at top and tensile at bottom near the centre zone of the plate.

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